Student Report

Experiments in Acoustical Refraction and Diffraction

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December 2, 2010

1 Abstract

In this experiment several measurement about acoustical waves were made. First we measured the speed of sounds in Aluminum, Copper and PVC with two different Methods. The Refraction at a 90° corner gives us the results which are shown in Table 3. The Time of Flight Method is seen in table 4. Both methods deliver reliable values. The literature values fit in the one sigma range. Furthermore the acoustical diffraction grating was analyzed and a single slit experiment was made. The same behaviour as for optical waves was expected but the data could not prove the theory. Last but not least the transmission of acoustical waves through thin plates at different angles were examined. For the two thinniest plates 2 peaks were observed, one for the longitudinal and one for the shear wave. The behaviour is the same as in a Fabry-Perot interferometer.

2 Theory

2.1 Refraction at a 90 degree corner

Looking for the maximas according to the different angles gives you two solutions. The smaller angle is for the longitudinal wave and the bigger angle for the shear wave. When you set up the experiment like it is shown in Figure 1 then you can easily apply snell's law to get the speed of sound in the different solids. The incident angle θ_w is measured and θ_s is 45°.

$$\frac{\sin\theta_w}{v_w} = \frac{\sin\theta_s}{v_s} \tag{1}$$

The error of v_s is shown in the appendix.
We use the same formular for the shear wave.

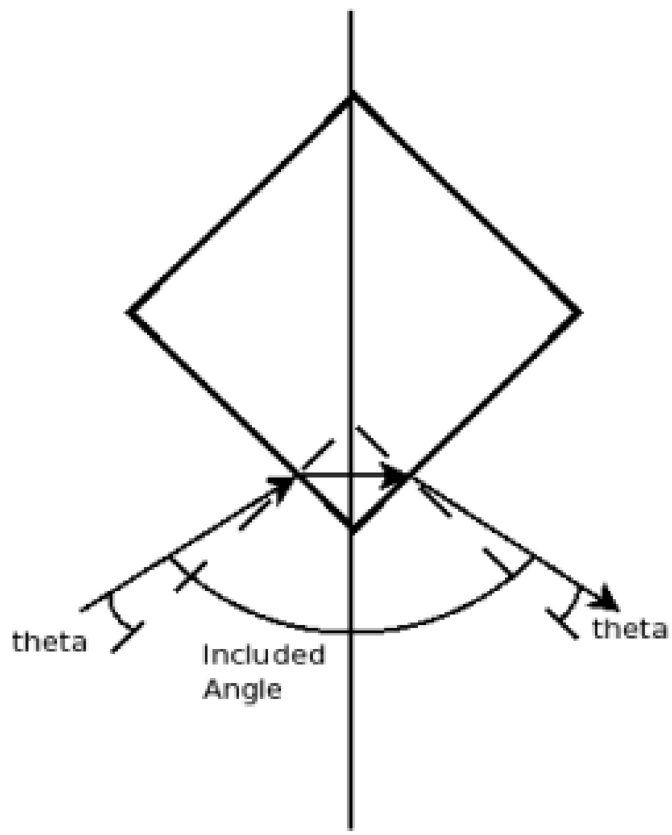

Figure 1: Setup of the refraction at a 90° corner

2.2 Time of Flight Method

We measure the time difference Δt of the recieved peak when we move the material along the axis for the distance d. Basic geometry yields to:

$$\Delta t = \frac{d\sqrt{2}}{\cos\theta_w}\left(\frac{\sin(45+\theta_w)}{v_s} - \frac{1}{v_w}\right) \tag{2}$$

Knowing the speed of sound in water and solving this equation for v_s gives us an independent result of the speed of sound in solids.

2.3 An Acoustical Defraction Grating

An acoustical wave is sent out on a diffraction grating on an aluminum solid. One use frequencys between 9MHz and 16MHz limited to the source transducer. Outside of that range the response is very low and there is a lot of noise. Now the reciever can be moved at various angles and an interference pattern can be found. The theory is:

$$d = g(\cos(90° - \alpha) - \cos(90° - \beta)) \tag{3}$$

where d is the path difference, α the incident angle, β the refraction angle and g the grating constant. For constructive interference the path difference has to be a multiple of λ and $\lambda = \frac{v}{f}$.

2.4 Single Slit Diffraction

The acoustical Single Slit Diffraction behaves like the optical one. The intensity of a single slit is proportional to $\left(\frac{\sin(x)}{x}\right)^2$ where $x = \pi\frac{b}{\lambda}\theta$. This is approximately:

$$(n + \frac{1}{2})\lambda = b\sin(\theta) \tag{4}$$

where b is the width of the single slit and λ the wavelength and n an integer.

2.5 Transmission Through Thin Plates

In theory the transmission of acoustical waves through thin plates is like a transmission of a light beam through a Fabry-Perot interferometer. There are transmission peaks when the path length of the acoustical waves in the solid is a multiple of the wavelengths. By knowing the speed of sound in water and the solid as well as the thickness b and the incident angle α you can calculate the path length d:

$$d = \frac{b}{\cos\beta} \text{where } \beta \text{ is the transmission angle} \tag{5}$$

$$\beta = \arcsin\left(\frac{v_s}{v_w}\sin\alpha\right) \tag{6}$$

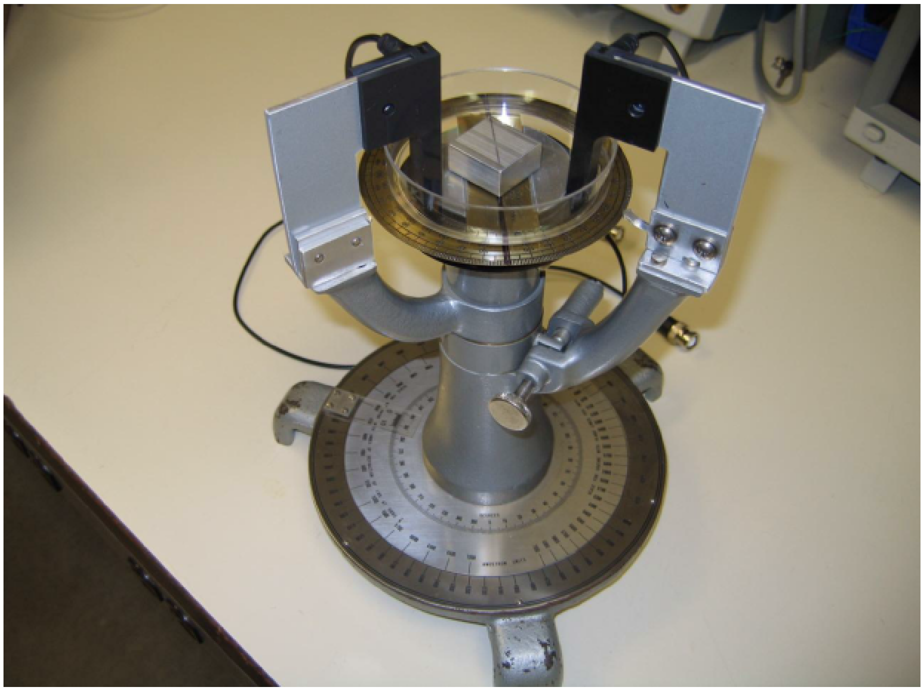

Figure 2: Setup

3 Experimental Details

3.1 Refraction at a 90 degree corner

The experiment was set up like in Figure 2. An acoustic device sends out acoustic pulses. The source of acoustic waves sends them out in the water with the solid sample in it. On the other side is an acoustical receiver, which can be moved at different angles, so you can adjust it for the maximum signal, which is the total angle of deflection. The solid sample is placed in the container of water so that its diagonal align with the line on the slab. Only in this case the scale of angles is calibrated.

3.2 Time of Flight Method

The setup of the Time of Flight Method is the same as the Refraction at a 90 degree corner. In this case you already know the maximum angles and you move the water container along the line with a distance measurement device, which tells you immediately the distance d. The time difference can be measured with the oscilloscope.

Frequency	Incident angle α	Reflection angle β	Grating Constant
9MHz	30	9	$4.8E-4m \pm 0.5E-4m$
13MHz	30	1	$4.8E-4 \pm 0.3E-4m$
16MHz	10	21	$5.065E-4m \pm 0.9E-4m$

Table 1: second Data of the Acoustical Diffraction Grating

3.3 An Acoustical Diffraction Grating

The setup of the experiment is basically the same as the setup of the Refraction at a 90° corner. I hold the incident angle $\alpha = 30°$ fixed, replaced the solids with the one with the diffraction grating and moved the acoustical receiver at various angles ϕ, looking for maxima. Furthermore I applied a sin wave of f=13MHz and f=12MHz with an amplitude of 10V. For f=13MHz maxima were found at $\phi = 26°$ and $\phi = 42°$ with amplitudes of A=23mV and A=13mV. For f=12MHz the maxima lay at $\phi = 12°, \phi = 12°, \phi = 27°$ and $\phi = 44°$. The experiment was done again and this time the data was much better. I took the angles of the first maximum of $f_1 = 9MHz$ and $f_3 = 16MHz$ and of the second maximum of $f_2 = 13MHz$. The corresponding angles are seen in table 1.

3.4 Single Slit Diffraction

A thin plate with a single slit was placed in the water, so that the transmitter was perpendicular to the plate and transmitted the acoustical waves directly through the single slit. On the other side the receiver was moved around at different angles and the angles and amplitudes of the maxima were recorded. This was done for two frequencys $f_1 = 16MHz$ and $f_2 = 10MHz$. For both frequencys the amplitude was 10V.
For f_1 maxima were found at $\phi = 0, 26°, 41°, -26°, -41°$ the corresponding amplitudes were A= 30mV, 4.1mV, 2.2mV, 4.2mV, 2.4mV.
For f_2 we got $\phi = -4°, 4°, 23°, 40°, 53°$ with amplitudes of A=32mV, 36mV, 13mV, 8mV and 9mV. It was very difficult again to identify the maximas correctly. For the second frequency it was astonishing that the main maximun was not be found at 0°, however at the two angles -4° and 4°.

3.5 Transmission Through Thin Plates

Different thicknesses of plates are provided which are placed along the line in the container of water. The acoustical transmitter and the acoustical receiver are facing each other. Then the container of water with the thin plate in it is turned and the amplitudes of the different angles are measured. The density of the thin plates is $7.89\frac{g}{cm^3}$. The token data can be seen in table 2.

Thickness	Amplitude	Angle
0.015”	70	0
	103	16
	246	33
0.010	76	0
	128	20
	198	34
0.002”	90	0
	430	58
0.005”	48	0
	250	49

Table 2: Data Results of Transmission Through Thin Plates

Material	Longitudinal wave	Shear wave	Textbook value
Aluminum	6300 ± 600	3020 ± 150	6420
Copper	4600 ± 400	2250 ± 80	4760
PVC	2370 ± 90	-	2380

Table 3: Results of the 90° corner refraction. Calculations are seen in Figure 3

4 Results and analysis

4.1 Refraction at a 90 degree corner

Based on snell's law and the propagation of errors, assuming an error on the angle of 1°, we got the speed of sounds in different samples. See calculations and table 4.

4.2 Time of Flight Method

According to formular 2 we can calculate the speed of sounds in the different solids. It were 3 data sets taken for each material and the mean value gives us the final result. I chose the mean value. Despite you make different errors for each run it fits the best in my opinion, because you cannot weight the errors, so you cannot use the weighted average. For this part of the experiment only the longitudinal wave was measured. This yields to:

4.3 An Acoustical Diffraction Grating

The various angles of maxima do not show much consistency. However, the grating constant can be calculated from specular reflection of the angles $\phi = 26°$ for f=13MHz and $\phi = 27°$ of f=12MHz. Using equation 3 yields to $g_{13MHz} = 1.86E - 3m \pm 2E - 5m$ and $g_{12MHz} = 2.71E - 3m \pm 2E - 5m$. The grating

Material	Longitudinal wave	Textbook value
Aluminum	6500 ± 200	6420
Copper	4710 ± 130	4760
PVC	2370 ± 30	2380

Table 4: Results of the Time of Flight Method. Calculations are shown in Figure 3

constants should be the same, thus there must be a big source of error.
Although I basically did the same the results were more consistent and prove the theory. The results are shown in table 1.

4.4 Single Slit Diffraction

It was expected that the maxima approximately appear at a multiple of angles. This could not be varified by assuming an error of 1° of the angle. Furthermore one has to figure out of which order the maxima are. If one consider the $\phi = 26°$ and $\phi = 23°$ as the first order one gets a single slit width of (according to equation 4): $b_{f_1} = 3.20E - 4m \pm 1.1E - 5m$ and $b_{f_2} = 5.8E - 4m \pm 2E - 5m$. This result does not match with the theory, because b should be around the same.

4.5 Transmission Through Thin Plates

Using equation 5 the distance d should be able to calculate, but because of the arcsin the distance d is not defined for angles bigger 33.9°. So this formular doesn't work in our case. It can be expected that you get two peaks for the longitudinal wave and for the shear wave. If the plate is too thin the shear waves cannot be observed. In the appendix a graph of amplitude vs angle is included on how it is supposed to look like.

5 Conclusion

5.1 Refraction at a 90 degree corner

The experiment is very susceptible for errors. First you have to align the solid material on the line which has to be as good as possible because otherwise the propagation of errors is huge. The water in the container makes it more difficult because it is more likely that the solid slides away. Furthermore the scale is hard to read, causing an error of 1 degree. However, all the results were in a one sigma range comparing to the textbook values. Unfortunatley I couldn't find any values for the shear wave, but they are expected to be approximatley half of the longitudinal wave which is true.

5.2 Time of Flight Method

The Time of Flight Method to measure the speed of sound in solids is based on geometrical constructions. The error on the distance d or the time difference Δt is rather small compared to the error you make if the setup is not symmetrical. It is very important that you move the container of water along the line that the setup remains symmmetrical. The results match pretty well with the textbook value and the speed of sound from the refraction at a 90 degree corner. It can be said that both the refraction at the corner and the Time of Flight Method are ways to get an estimate about the speed of sounds in solids. However, it is limited to the ability to get the geometry right.

5.3 An Acoustical Diffraction Grating

The identification of the maxima was very uncertain. It was hard to see if it is a maxima or not. Thus the error is probably bigger than the assumed 1°. It might be also an error of the setup of the experiment. In this case and in the Single Slit Diffraction I used the pulse generator. Actually the setup seemed to be right, because every adjustment of the setting could be seen on the oscilloscope. To improve the experiment frequencys at the upper and lower limit of the source transducers should be chosen. After redoing the experiment the given grating constant $g = 0.508mm$ fits within a one sigma range of the calculated results. Thus the theory is proved. The assumend error were still 1° of α and β.

5.4 Single Slit Diffraction

The theory could not be varified with the measured data. Possible errors which could have occured are additional scattering, wrong settings, the theory is only an approximation. To improve the experiment I would suggest to measure the minimas, because there the theory is simple and no approximations have been used. However it is not easy to identify the minima. Furthermore one will get more accurate results if one reduces the single slit width.

6 Appendices

6.1 Error Calculations

$$s_{v_s} = v_s \sqrt{\left(\frac{\cos\theta_s \pi}{180 \sin\theta_s}\right)^2 + \left(\frac{\cos\theta_w \pi}{180 \sin\theta_w}\right)^2} \quad (7)$$

Refraction at a 90 degree corner

	Longitudinal			Shear						
	Angle left	Angle right	Incident angle	Angle left	Angle right	Incident angle	v_long[m/s]	v_error_long	v_shear[m/s]	v_error_shear
Aluminum	54,5	125	9,75	65	114	20,5	6250,62	644,19	3022,61	150,64
Copper	58,5	122	13,25	72	106	28	4618,4	351,68	2254,75	83,82
PVC	70	107	26,5	x	x	x	2372,35	92,8	x	x

Time of Flight Method

	T_1 [µs]	T_2 [µs]	ΔT [µs]	D [mm]	Theta [°]	Theta [rad]	v_long	v_average
	30,16	24	6,16	8,01	9	0,16	6180,18	
	26,56	18,64	7,92	10,04	9	0,16	6910,38	
Aluminum	28,24	23,6	4,64	6	9	0,16	6325,1	6471,89
	25,68	20,48	5,2	7,35	13	0,23	4696,81	
	26,16	20,8	5,36	7,7	13	0,23	4501,35	
Copper	27,76	22,64	5,12	7,11	13	0,23	4934,62	4710,93
	27,2	24,88	2,32	5,37	26,5	0,46	2403,2	
	27,12	23,76	3,36	7,77	26,5	0,46	2404,76	
PVC	27,6	24,08	3,52	8,71	26,5	0,46	2300,3	2369,42

(v_long-v_ave	sum	varianz	error_v_aver
85094,71			
192278,36			
21546,01	298919,08	386,6	223,2
199,36			
43920,51			
50037,9	94157,77	216,98	125,27
1141,41			
1248,74			
4777,9	7168,05	59,87	34,56

Figure 3: Calculations for Refraction at a 90 degree corner and Time of Flight Method

$$S = \sqrt{S^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2} \tag{8}$$

$$s_{\bar{X}} = \frac{S}{\sqrt{n}} \tag{9}$$

$$s_g = g\frac{\pi}{180°(\sin\alpha - \sin\beta)}(\cos\alpha + \cos\beta) \tag{10}$$

$$s_b = (n + 1/2)\frac{\lambda\cos\phi}{\sin^2\phi}\frac{\pi}{180°} \tag{11}$$

6.2 Graphs and Pictures

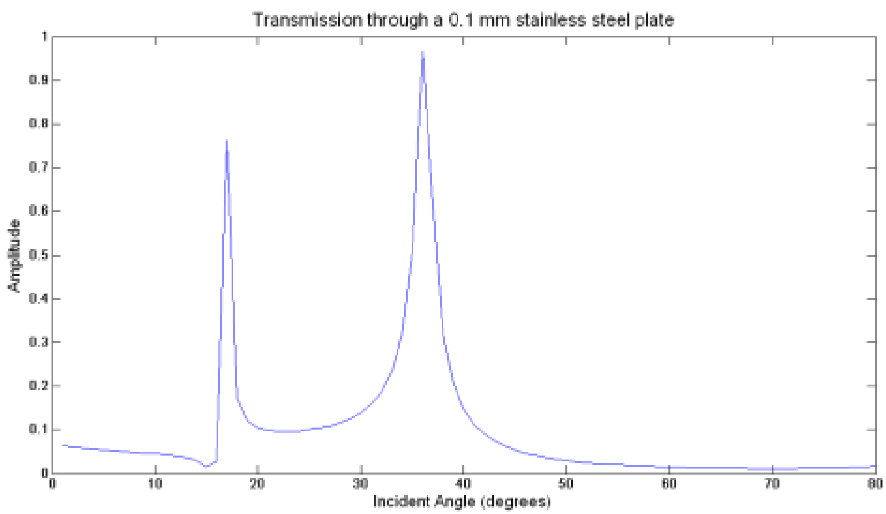
Transmission through a 0.1 mm stainless steel plate
Amplitude
Incident Angle (degrees)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70
80

Figure 4: